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The Impact of Promotional Activities and Inflationary Trends on a Deteriorated Inventory Model Allowing Delay in Payment

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Abstract A deteriorating inventory model allowing delay in payment, promotional activities and inflation is developed. This arises as a result of the time gap in between the time of estimation and the starting time of economic order quantity (EOQ) system and promotional activities with a permissible delay in payment will affect the inventory total cost. Moreover, the political volatility of a country leads to a much more unstable situation in the present world economy. So a change in inflation takes place. The objective of this inventory model is to minimize the total inventory cost allowing promotional activities and delay in payment for deteriorating items under inflation with shortages. The numerical analysis shows that an appropriate policy can be benefited the retailer for deteriorating items. Finally, sensitivity analysis of optimal solution with respect to the major parameters are also studied.

Keywords: *inventory model, deterioration, promotion, inflation, delay in payment, EOQ, shortages*

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1. Introduction

Inventory model is an important part of operations research, which may be used in variety of problems. To make it applicable in real life situation researchers are engaged in modifying existing models on different parameters under various circumstances. Inventories play a major role in the united stages economy and have been in excess of 22% of the nation's gross national product over the past few decades. As millions of dollars are tied up in inventories, proper management of these inventories can lead the organisation or company to become profitable in the entire globe. A major concern of inventory management is to know when and how much to order or manufacture so that the total cost per unit time is minimized. The total cost consists of carrying, shortage, replenishment or setup cost and the purchase or production cost. In the development of an EOQ system, we usually omit the case of inflation of money, promotional activities and deterioration of the items like fruit, milk, drug, chemicals, gasoline etc. But in the real competitive world they exist and are quite flexible in nature. On the other hand to attract customer to order more quantities, usually suppliers offer a certain credit period without interest during the permissible delay time period. Inventory management plays a significant role for production system in business since it can help companies reach the goal of ensuring prompt delivery, avoiding shortages, helping sales at competitive prices and so forth for achieving competitive advantage. However, excessive simplification of assumptions results in mathematical models that do not represent the inventory situation to be analyzed.

The classical analysis builds a model of an inventory system and calculates the EOQ which minimize the costs satisfying minimization criterion. One of the unrealistic assumption is that items stocked preserve their physical characteristics during their stay in inventory for long run. Items in stock are subject to many possible risks, e.g. damage, spoilage, dryness, vaporization etc., those results decrease of usefulness of the original one and a cost is incurred to account for such risks of the product.

The problem of deteriorating inventory has received considerable attention in recent years. This is a realistic trend since most products such as medicine, diary products and chemicals starts to deteriorate once they are produced.

Most researches in inventory do not consider inflation with promotional activities. This is unrealistic, since the resource of an enterprise depends very much on when it is used and this is highly correlated to the return of investment. Therefore, taking into account the inflation of money should be critical especially when investment and forecasting are considered.

Buzacott (1975) [1] explained inventory model to determine economic order quantities with inflation. Datta and Pal (1991) [2] verified the effect of inflation and time value of money on an inventory model with linear time dependant demand rate and shortages. Haneveld and Teunter (1998) [3] discussed the inventory model where they studied the effects of discounting and demand rate variability on the EOQ. Holland (1995) [4] derived an inventory model where inflation and uncertainty tests for temporal ordering are allowed. Horowitz (2000) [5] presented an inventory model for obtaining EOQ under uncertain inflation of the economy. Liao et al. (2000) [6] derived an inventory model with deteriorating items under inflation when a delay in payment is permissible. Mishra (1979) [7] focused an optimal inventory management policy under inflation. Moon and Lee (2000) [8] verified the effect of inflation and time value of money on an EOQ order quantity model with random product life cycle. Pattnaik, M. (2012) [9] and Pattnaik (2014) [10] derived different types of deterministic inventory models for deteriorating items in finite horizon. Shah (1993) [11] discussed a lot size EOQ model for exponentially decaying inventory when delay in payments is permissible. Skouri and Papachristors (2002) [12] developed deterministic inventory lot-size models under inflation with shortages and deterioration for the fluctuating demand by Yang et al. Their approach was performed in the framework of the total inventory cost under inflation criterion.

The objective of this paper is to minimize the inventory cost so as to determine the optimal order quantity and promotional effort factor under inflation where shortages are allowed. The remainder of paper organized in section 2 is assumptions and notations for development of the model. The mathematical model is developed in section 3. Optimization is given in section 4. Numerical example is presented to illustrate the development of model in section 5. The sensitivity analysis carried out in section 6 to observe the changes in optimal solution. Finally section 7 deal with conclusion.

2. Assumption and Notation

The mathematical model in this paper is developed with the following assumptions:

1. The replenishment is instantaneous.
2. The lead time is zero.
3. Backlogging is allowed within a small interval of time.
4. The total demand for the whole planning period is unknown and constant.
5. Discounts are not allowed and a permissible delay period will not exceed the length of a cycle.
6. During the fixed credit period *M*, a deposit is made from the unit cost of generated sales revenue into an interest-bearing account. The delay expenses of the system can be overcome from the difference between retail price and unit cost. At the end of period *M*, the account is settled, and interest changes are payable on the account in stock.

Notations

The following notations are used

T: Time period of each cycle

t_1: Time from where shortage begins

M: Permissible delay period for settling accounts

H: Length of the planning horizon ($H=mT$, $m>1$ is a positive integer)

$I_1(t)$: Inventory level at any time $0 \le t \le t_1$

$I_2(t)$: Inventory level at any time $t_1 \le t \le T$

Q: Order size per cycle

D: Total demand per year

S: Shortage quantity per cycle

K: Rate of inflation ($/unit time)

Θ: Deterioration rate (<1) per unit time

H: Holding cost per unit/year

S_0: Unit shortage cost per unit time

$C(t)$: Unit purchasing price for an item bought at time t, where c(t)=c0(1+kt), 0≤t≤(m-1)t, c0 is the unit price of the item at time zero

$A(t)$: Ordering cost for an order placed at time t, i.e. $A(t)=A_0(1+kt)$, $0\le t\le (m-1)t$, A_0 is unit ordering cost at time zero

i_e: Annual interest that can be earned per unit

i_c: Annual interest charges payable per until where $i_c>i_e$.

P: The promotional effort factor per cycle.

$PE(\rho)$: The promotional effort cost, $PE=(\rho)=K_1(\rho-1)^2[d(s)]^{\alpha_1}$, where, $K_1>0$ and α_1 is a constant.

3. Mathematical Model

The total period *H* is divided into m equal sub-intervals of constant length *T* such that *H-mT*. For a subinterval $(0,T)$ the inventory *Q* is received at time $t=0$. Due to demand and deterioration, the inventory level reaches zero level at time $t=t_1$ and shortages start to occur. The shortage continue upto $t=T$ and accumulates to *S* quantity. If $I_1(t)$ and $I_2(t)$ denotes the inventory lee at any time *t* where $0 \le t \le t_1$ and $t_1 \le t \le T$ respectively. Figure 1 represents the inventory level for delay in payment. Then the differential equation

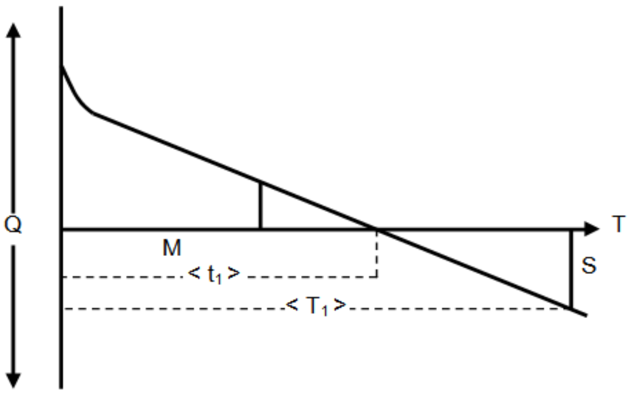

Figure 1. Inventory Level for Delay in Payment

$$\frac{dI_1(t)}{dt}+\theta I_1(t)=-D \quad 0\le t_1 \tag{1}$$

$$\frac{dI_{21}(t)}{dt} = -D \qquad t_1 < t \le T \tag{2}$$

With

$$\mathrm{I_1(0) = Q - S}\ ,\mathrm{I_1(t_1) = I_2(t_1) = 0}\ \mathrm{I_2(T) = -S} \tag{3}$$

Solving equation(1) and (2)

Equation (1) is linear differential equation.

Now Integrating factor $e^{\int \theta dt} = e^{\theta t}$

Multiplying both side of equation (1) by Integrating factor we get

$$\frac{d}{dt} I_1(t) e^{\theta t} = -De^{\theta t}$$

$$\Rightarrow I_1(t) e^{\theta t} = \int De^{\theta t} dt = -\frac{De^{\theta t}}{\theta} + C \tag{4}$$

$$\Rightarrow I_1(t) = -\frac{D}{\theta} + Ce^{-\theta t}$$

By using initial condition $I_1(t_1) = 0$

From equation (4) $0 = \frac{-D}{Q} + Ce^{-\theta t t_1}$

$$\Rightarrow Ce^{-\theta t t_1} = \frac{D}{Q}$$

$$\Rightarrow C = \frac{De^{\theta t t_1}}{\theta}$$

So

$$\mathrm{I_1(t)} = \frac{-D}{\theta} + \frac{De^{\theta t_1}}{\theta} e^{-\theta t} = \frac{-D}{\theta} + \frac{De^{\theta(t_1 - t)}}{\theta}$$
$$\Rightarrow I_1(t) = \frac{D}{\theta}\left[e^{\theta(t_1 - t)} - 1\right] \tag{5}$$

From equation (2)

$$\frac{D\,I_2(t)}{dt} = -D$$

$\Rightarrow \quad \int dI_2(t) = -\int D.dt$

$\Rightarrow \quad I_2(t) = -Dt + C_1$

From initial condition $I_2(t_1) = 0$ when $t = t_1$

$O = -Dt_1 + C_1$

$\Rightarrow \quad C_1 = Dt_1$

Hence the solution is,

$$\mathrm{I_2(t) = -Dt + Dt_1 = D(t_1 - t)} \tag{6}$$

Shown in appendix A

$$I_1(0) = Q - S \approx D\left(t_1 + \frac{\theta t_1^2}{2}\right) \tag{7}$$

and

$$t_1 \approx \frac{Q-S}{D} - \frac{\theta(Q-S)^2}{2D^2} = T - \frac{S}{D} \tag{8}$$

(as $\theta \le 1$ *neglecting* $0(\theta^2)$ and higher order)

Case-1: M<t₁

As A(t) = replenishment cost at time t

C(t) = Purchasing cost at time t

C_r = Replenishment cost in (O,H)

$C_r = \sum_{n=0}^{m-1} A(nT)$ Given that

$0 \le t \le (m-1)T\ ,\ A(t) = A_0(1+kt)$

$$C_r = \sum_{n=0}^{m-1} A_0(1+knT) = A_0 \sum_{n=0}^{m-1}(1+KnT)$$
$$= A_0\left[\sum_{n=0}^{m-1} 1 + \sum_{n=0}^{m-1} KnT\right]$$
$$\therefore \quad = A_0\left[\begin{pmatrix} 0+1+1+........ \\ upto\ m-1 \end{pmatrix} + KT\left(0+1+2+......+(m-1)\right)\right] \tag{9}$$
$$= A_0\left[m + KT\frac{(m-1)(m)}{2}\right]$$
$$= A_0 m\left[1 + \frac{KT}{2}(m-1)\right]$$

The purchasing cost C_p in (O,H) is

$$\mathrm{C_p} = Q\sum_{n=0}^{m-1} C(nT)$$

Given C(t) = C_0 (1+Kt) where 0<t < (m–1) T

$$\therefore \mathrm{C_p} = Q\sum_{n=0}^{m-1} C_0(1+KnT) = QC_0\sum_{n=0}^{m-1}(1+KnT)$$
$$\therefore \mathrm{QC_0}\left[\sum_{n=0}^{m-1} 1 + \sum_{n=0}^{m-1} KnT\right]$$
$$\mathrm{QC_0}\left[m + KT.\frac{(m)(m-1)}{2}\right] = mQC_0\left[1 + \frac{kT(m-1)}{2}\right]$$
$$\therefore \qquad C_p = mC_0 Q\left[1 + \frac{kT}{2}(m-1)\right] \tag{10}$$

The holding cost in (O,H)

$$\mathrm{C_h} = h\sum_{n=0}^{m-1} C(nT)\int_0^{t_1} I_1(t)\,dt$$
$$= \frac{mC_0 h(Q-S)^2}{2D}\left(1 - \frac{2\theta(Q-S)}{3D}\right)\left[1 + \frac{KT}{2}(m-1)\right] \tag{11}$$

The shortage cost in (O,H) is

$$\mathrm{C_s} = S_0\sum_{n=0}^{m-1} C(nT + t_1)\int_{t_1}^{T} I_2(t)\,dt$$

But given C(t) = C_0 (1+kt)

$$0 \le t \le (m-1)T$$

$$= S_0\sum_{n=0}^{m-1} C(nT + t_1)\int_{t_1}^{T} D(t_1 - t)dt$$
$$= S_0\sum_{n=0}^{m-1} C(nT + t_1)\int_{t_1}^{T} (Dt_1 - Dt)dt$$

$$= S_0 \sum_{n=0}^{m-1} C(nT+t_1)\left[Dt_1 t - \frac{Dt^2}{2}\right]_{t_1}^{T}$$

$$= S_0 \sum_{n=0}^{m-1} C(nT+t_1)\left(Dt_1 T - Dt_1^2 - \frac{DT^2}{2} + \frac{Dt_1^2}{2}\right)$$

$$= S_0 \sum_{n=0}^{m-1} C(nT+t_1)\left(Dt_1 T - \frac{Dt_1^2}{2} - \frac{DT^2}{2}\right)$$

$$= S_0 \sum_{n=0}^{m-1} C(nT+t_1).D\left(t_1 T - \frac{t_1^2}{2} - \frac{T^2}{2}\right)$$

$$= D\left(t_1 T - \frac{t_1^2}{2} - \frac{T^2}{2}\right) S_0 \sum_{n=0}^{m-1} C_0\left(1+k\left(nT+t_1\right)\right)$$

$$= D\left(t_1 T - \frac{T^2}{2} - \frac{t_1^2}{2}\right) S_0 \; C_0 \sum_{n=0}^{m-1} C_0\left(1+knT+kt_1\right)$$

$$= D\left(Tt_1 - \frac{T^2}{2} - \frac{t_1^2}{2}\right) S_0 C_0 \left(m + \frac{KT(m)(m-1)}{2} + kt_1 m\right)$$

$$= \frac{mDC_0S_0}{2}\left(2Tt_1 - T^2 - t_1^2\right)\left(1+\frac{KT(m-1)}{2} + kt_1\right)$$

$$= \frac{-mDC_0S_0}{2}\left(t_1 - T\right)^2\left(1+kt_1+\frac{KT(m-1)}{2}\right)$$

$$= \frac{-mDC_0S_0}{2}\left(\frac{S}{D}\right)^2\left(1+kt_1+\frac{KT(m-1)}{2}\right)$$

$$\therefore\; t_1 = T - \frac{S}{D}$$

$$\therefore\; C_s = +\frac{mC_0S_0S^2}{2D}\left(1+kt_1+\frac{KT}{2}(m-1)\right) \qquad (12)$$

The interest charged for the inventory not being after the due date M in (0,H) is

$$C_i = i_c \sum_{n=0}^{m-1} C(nT)\int_M^{t_1} I_1(t)\,dt$$

$$= \frac{i_e m C_0}{2}(Q-S-MD)\left[\begin{array}{c}\dfrac{Q-S-MD}{D} - Q \\ \left\{\begin{array}{c}\dfrac{2(Q-S)(Q-S+MD)}{3D^2} \\ -\dfrac{M^2}{3}\end{array}\right\}\end{array}\right] \qquad (13)$$

$$\left\{1+\frac{KT}{2}(m-1)\right\}$$

Detail calculation of C_h and C_i are given in appendix A. Interest earned in (O,H) is

$$C_{e_1} = i_e \sum_{n=0}^{m-1} C(nT)\int_0^M D\,dt$$

$$= i_e \sum_{n=0}^{m-1} C(nT)\int_0^M D\left[t\right]_0^M = i_e \sum_{n=0}^{m-1} C(nT)DM$$

$$= i_e DM \sum_{n=0}^{m-1} C_0(1+KnT) = i_e DMC_0\left[m+\frac{KT(m)(m-1)}{2}\right]$$

$$C_{e_1} = i_e mMDC_0\left[1+\frac{KT(m-1)}{2}\right] \qquad (14)$$

From equation (9) – (14) the total system cost over (0,H) is given by

$$\mathrm{TC_1(Q,T) = C_r + C_p + C_h + C_s + C_{pe} + C_i - C_{ei}}$$

$$= \mathrm{m[F_{11}(Q,S) - QF_{12}(Q,S) + KF_{13}(Q,S - QK\ F_{14}(Q,S)]}$$

$$TC_1(Q,T) = A_0 m\left[1+\frac{KT}{2}(m-1)\right]$$

$$+mC_0Q\left[1+\frac{KT}{2}(m-1)\right]$$

$$+\frac{mC_0h(Q-S)^2}{2D}\left(1-\frac{2\theta(Q-S)}{3D}\right)$$

$$\left(1+\frac{KT}{2}(m-1)\right)+\frac{mC_0S_0S^2}{2D}\left(1+Kt_1+\frac{kT}{2}(m-1)\right)$$

$$+k_1(\rho-1)^2 r^{\alpha_1} + K_1(\rho-1)^2 d(s)^{\alpha_1}$$

$$+\frac{i_c mC_0}{2}(Q-S-MD)$$

$$\left[\frac{Q-S-MD}{D} - \theta\times\left\{\frac{2\times(Q-S)(Q-S-MD)}{3D^2} - \frac{M^2}{3}\right\}\right]$$

$$\left\{1+\frac{KT}{2}(m-1)\right\} - \left[i_e mMDC_0\left(1+\frac{KT}{2}(m-1)\right)\right]$$

$$= mA_0\left[1+\frac{KT}{2}(m-1)\right] + mC_0Q\left[1+\frac{KT}{2}(m-1)\right]$$

$$+\frac{mC_0h(Q-S)^2}{2D}\left(1+\frac{KT}{2}(m-1)\right) - \frac{mCoh(Q-S)^3\theta}{3D^2}$$

$$\left(1+\frac{kT}{2}(m-1)\right)+\frac{mC_0S_0S^2}{2D}Kt_1 + \frac{mC_0S_0S^2}{2D}$$

$$\left(1+\frac{KT}{2}(m-1)\right)+K_1(\rho-1)^2 d(s)^{\alpha_1}$$

$$+\frac{i_c mC_0}{2D}(Q-S-MD)^2\left(1+\frac{KT}{2}(m-1)\right)$$

$$-\frac{i_e MC_0}{2}\theta(Q-S-MD)\left\{2\frac{(Q-S)(Q-S+MD)}{3D^2} - \frac{M^2}{3}\right\}$$

$$\left\{1+\frac{KT}{2}(m-1)\right\} - i_e mMDC_0\left(1+\frac{KT}{2}(m-1)\right)$$

$$= mA_0 + mC_0Q + \frac{mC_0h(Q-S)^2}{2D} - \frac{mCoh(Q-S)^3}{3D^2}\theta$$

$$+\frac{mC_0S_0S^2}{2D} - \frac{i_c mC_0}{2D}(Q-S-MD)^2 - i_e mMDC_0$$

$$-\frac{i_e m\theta MC_0}{6}\begin{pmatrix}Q-S \\ -MD\end{pmatrix}\left\{2\frac{(Q-S)(Q-S+MD)}{D^2} - M^2\right\}$$

$$+K_1(\rho-1)^2 d(s)^{\alpha_1} + mA_0\frac{KT}{2}(m-1) + mC_0Q\frac{kT}{2}(m-1)$$

$$+\frac{mC_0h(Q-S)^2}{2D}\frac{KT}{2}(m-1)$$

$$-\frac{mCoh(Q-S)^3\theta}{3D^2}\frac{kT}{2}(m-1)\theta$$

$$+\frac{m_0C_0S_0S^2}{2D}\frac{kT}{2}(m-1)+\frac{i_cmC_0}{2D}\begin{pmatrix}Q-S\\-MD\end{pmatrix}^2\frac{kT}{2}(m-1)$$

$$-\frac{i_em\theta MC_0}{6}(Q-S-MD)$$

$$\left\{2\frac{(Q-S)(Q-S+MD)}{D^2}-M^2\right\}\frac{kT}{2}(m-1)$$

$$-i_emMDC_0\frac{kT}{2}(m-1)+\frac{m_0C_0S_0S^2}{2D^2}k(TD-S)$$

$$\Rightarrow TC_1=m\begin{bmatrix}A_0+C_0\begin{Bmatrix}Q+\frac{h(Q-S)^2}{2D}+\\\frac{S_0S^2}{2D}+\frac{i_c(Q-S-MD)^2}{2D}\end{Bmatrix}\\-i_eMD\end{bmatrix}$$

$$-m\theta C_0\begin{bmatrix}\frac{h(Q-S)^3}{3D^2}+\frac{i_c(Q-S-MD)}{6}\\\left(2\frac{(Q-S)(Q-S-MD)^2}{D^2}-M^2\right)\end{bmatrix}+\frac{mkT}{2}(m-1)$$

$$\begin{bmatrix}A_0+C_0\begin{pmatrix}Q+\frac{h(Q-S)^2}{2D}+\frac{S_0S^2}{2D}\\+\frac{i_c(Q-S-MD)^2}{2D}\\-i_eMD\end{pmatrix}+\frac{C_0S_0S^2(T(D-S)}{TD^2(m-1)}\\\Rightarrow Ce_2=mMC_0Di_e\left[1+\frac{KT}{2}(m-1)\right]\qquad\therefore t_1=M\end{bmatrix}$$

$$-\frac{m\theta kC_0T}{2}(m-1)\begin{bmatrix}\frac{h(Q-s)^3}{3D^2}+\frac{i_c(Q-S-MD)}{6}\\\left(2\frac{(Q-S)(Q-S-MD)}{D^2}-M^2\right)\end{bmatrix}$$

$$\therefore TC_1(Q,T)=m\begin{bmatrix}F_{11}(Q,S)-\theta F_{12}(Q,S)\\+K\,F_{13}(Q,S)-\theta KF_{14}(Q,S)\end{bmatrix}$$

Where

$$F_{11}(Q,S)=A_0+C_0\begin{Bmatrix}Q+\frac{h(Q-S)^2}{2D}\\+\frac{S_0S^2}{2D}+\frac{i_c(Q-S-MD)^2}{2D}-i_eMD\end{Bmatrix}$$

$$F_{12}(Q,S)=C_0\begin{bmatrix}\frac{h(Q-S)^3}{3D^2}+\frac{i_c(Q-S-MD)}{6}\\\left(2\frac{(Q-S)(Q-S+MD)^2}{D^2}-M^2\right)\end{bmatrix}$$

$$F_{13}(Q,S)=\frac{(m-1)T}{2}\left[\frac{C_0S_0S^2(T(D-S)}{TD^2(m-1)}+F_{11}(Q,S)\right]$$

$$F_{14}(Q,S)=\frac{(m-1)T\,F_{12}(Q,S)}{2}$$

Case –II (M = t_1)

Thus using equation 9-12 and 16 we have the total system cost over (0,H)

$$TC_2(Q,T)=C_r+C_p+C_h+C_s+C_{pe}-Ce_2$$

$$=mA_0\left[1+\frac{KT}{2}(m-1)\right]+mC_0Q\left[1+\frac{KT}{2}(m-1)\right]$$

$$+\frac{mC_0h(Q-S)^2}{2D}\left(1-\frac{2\theta(Q-S)}{3D}\right)\left[1+\frac{kT}{2}(m-1)\right]$$

$$+\frac{mC_0S_0S^2}{2D}\left[1+kt_1+\frac{KT}{2}(m-1)\right]$$

$$-i_emMDC_0\left\{1+\frac{KT}{2}(m-1)\right\}+K_1(\rho-1)^2\,d(s)^{\alpha_1}$$

$$=mA_0+mC_0Q+\frac{mC_0h(Q-S)^2}{2D}\left[1+\frac{KT}{2}(m-1)\right]$$

$$+\frac{mC_0S_0S^2}{2D}\left[1+\frac{KT}{2}(m-1)\right]-i_eC_0mMD$$

$$-\frac{mC_0\theta h(Q-S)^3}{3D^2}\left[1+\frac{kT}{2}(m-1)\right]+\frac{mC_0S_0S^2}{2D}kt_1$$

$$-i_emMDC_0\frac{KT}{2}(m-1)+mA_0\frac{KT}{2}(m-1)$$

$$+mC_0Q\frac{KT(m-1)}{2}+K_1(\rho-1)^2\,d(s)^{\alpha_1}$$

$$=mA_0+mC_0Q+\frac{mC_0h(Q-S)^2}{2D}+\frac{mC_0S_0S^2}{2D}$$

$$-i_eC_0mMD-\frac{mC_0\theta h(Q-S)^3}{3D^2}+\frac{mC_0S_0S^2}{2D^2}KM$$

$$+\frac{mC_0h(Q-S)^2}{2D}\frac{KT(m-1)}{2}+\frac{mC_0S_0S^2}{2D}\frac{KT}{2}(m-1)$$

$$-\frac{mC_0\theta h(Q-S)^3}{3D^2}\frac{KT}{2}(m-1)-i_emMDC_0\frac{KT}{2}(m-1)$$

$$+mA_0\frac{KT}{2}(m-1)+mC_0Q\frac{KT}{2}(m-1)+K_1(\rho-1)^2\,d(s)^{\alpha_1}$$

$$-mA_0+mC_0Q+\frac{mC_0h(Q-S)^2}{2D}+\frac{mC_0S_0S^2}{2D}-i_eC_0mMD$$

$$-\frac{mC_0h(Q-S)^3}{3D^2}\theta+\frac{mC_0S_0S^2}{2D}M+\frac{mC_0h(Q-S)^2}{2D}$$

$$\times\ \frac{T}{2}(m-1)+\frac{mC_0S_0S^2}{4D}T(m-1)-i_emMDC_0\frac{T}{2}(m-1)$$

$$+\ mA_0\frac{T}{2}(m-1)+mC_0Q\frac{T}{2}(m-1)k-$$

$$\left[\frac{mC_0h(Q-S)^3T(m-1)}{6D^2}\times\theta k\right]$$

$$= m\left[\left(\begin{array}{l} A_0 + C_0 Q + \frac{C_0 h(Q-S)^2}{2D} \\ + \frac{C_0 S_0 S^2}{2D} - i_e C_0 D \end{array}\right)\right]$$

$$+ \frac{mC_0 S_0 S^2 T(m-1)}{4D} - \frac{i_e MDC_0 T(m-1)}{2} + \frac{A_0 T(m-1)}{2}$$

$$+ \frac{mC_0 QT(m-1)}{2}\}K - \left(\frac{C_0 h(Q-S)^3 T(m-1)}{6D^2}\right)\theta K$$

$$+ K_1(\rho-1)^2 d(s)^{\alpha_1}$$

$$= m\left[\begin{array}{l} \left(\begin{array}{l} A_0 + C_0(Q + \frac{h(Q-S)}{2D} \\ + \frac{S_0 S^2}{2D} - i_e MD \end{array}\right) - \left(\frac{C_0 h(Q-S)^3}{3D^2}\right)\theta \\ + \left[\frac{(m-1)T}{2}\right](\frac{C_0 S_0 S^2 M}{D(m-1)T} + C_0 Q \\ + A_0 + \frac{C_0 S_0 S^2}{2D} - i_e MDC_0 + \frac{C_0 h(Q-S)^2}{2D})]k \\ -(\frac{C_0 h(Q-S)^3}{3D^2}\theta k \frac{(m-1)T}{2}) \end{array}\right]$$

$$= m\left[\begin{array}{l} \left\{A_0 + C_0 Q + \frac{h(Q-S)^2}{2D} + \frac{S_0 S^2}{2D} - i_e MD\right\} \\ + \left(\frac{C_0 h(Q-S)^3}{3D^2}\right)\theta + \\ \left(\frac{(m-1)T}{2}\left(\begin{array}{l} \frac{C_0 S_0 S^2 M}{(m-1)TD} + A_0 + C_0 + \\ \left\{Q + \frac{S_0 S^2}{2D} - i_e MD + h\frac{(Q-S)^2}{2D}\right\} \end{array}\right)\right) k \\ + \left(\frac{(m-1)T}{2}\left(\frac{C_0 h(Q-S)^3}{3D^2}\right)\right)\theta K \end{array}\right]$$

$$+ K_1(\rho-1)^2 d(s)^{\alpha_1}$$

$$\therefore TC_2(Q,T) = m\left[\begin{array}{l} F_{21}(Q,S) - \theta F_{22}(Q,S) \\ + K F_{23}(Q,S) - \theta K F_{24}(Q,S) \end{array}\right]$$

$$+ K_1(\rho-1)^2 d(s)^{\alpha_1}$$

Where

$$F_{21}(Q,S) = \left[A_0 + C_0\left\{Q + \frac{h(Q-S)^2}{2D} + \frac{S_0 S^2}{2D} - i_e MD\right\}\right]$$

$$F_{22}(Q,S) = \frac{C_0 h(Q-S)^3}{3D^2}$$

$$F_{23}(Q,S) = \frac{(m-1)T}{2}\left[\frac{C_0 S_0 S^2 M}{(m-1)TD} + F_{21}(Q,S)\right]$$

$$F_{24}(Q,S) = \frac{(m-1)T\, F_{22}(Q,S)}{2}$$

Case-III $t_1 < M < T$

No interest is charged in (0,H) because the supplier can be paid in full at the time of permissible delay and hence the interest earned in (0,H) is

$$C_{e_3} = i_e \sum_{n=0}^{m-1} C(nT).\int_0^{t_1} D\,dt + (M - t_1)Dt_1$$

$$= i_e \sum_{n=0}^{m-1} C(nT).\{D\,dt + (M - t_1)Dt_1\}$$

$$= i_e \sum_{n=0}^{m-1} C(nT).\left\{\begin{array}{l} D\left(\frac{(DT-S)}{D}\right) + \\ \left(M - \frac{(DT-S)}{D}\right)D\left(\frac{(DT-S)}{D}\right) \end{array}\right\}$$

$$= i_e \sum_{n=0}^{m-1} C(nT).\left\{(DT-S) + \left(\frac{(MD-DT+S)}{D}\right)(DT-S)\right\}$$

$$= i_e \sum_{n=0}^{m-1} C(nT).\left\{(DT-S)\left(1 + \frac{(MD-DT+S)}{D}\right)\right\}$$

$$= i_e \sum_{n=0}^{m-1} C(nT).\left\{\frac{(DT-S)(D+MD-DT+S)}{D}\right\}$$

$$= i_e \frac{(TD-S)(D+S+MD-DT)}{D}\sum_{n=0}^{m-1} C_0(1+knT)$$

$$= i_e \frac{(TD-S)\left(\begin{array}{l} D+S \\ +MD-DT \end{array}\right)}{D} C_0\left\{m + \frac{kTm(m-1)}{2}\right\}$$

$$= \frac{i_e\, mC_0(TD-S)\left(\begin{array}{l} D+S \\ +MD-DT \end{array}\right)}{D}\left(1 + \frac{kT}{2}(m-1)\right)$$

Using C_r, C_ρ, $C_h^{'}, C_S$ and C_{ρ_e} and C_{e_3}

The total system cost over (0,H)

$$\mathrm{TC}_3(\mathrm{Q,T}) = \mathrm{C_r} + \mathrm{C_p} + \mathrm{C_h} + \mathrm{C_s} + \mathrm{C_{pe}} - \mathrm{Ce_3}$$

$$= mA_0\left[1 + \frac{KT}{2}(m-1)\right] + mC_0 Q\left[1 + \frac{KT}{2}(m-1)\right]$$

$$+ \frac{mC_0 h(Q-S)^2}{2D}\left(1 - \frac{2Q(Q-S)}{3D}\right)\left[1 + \frac{KT}{2}(m-1)\right]$$

$$+ \frac{mC_0 S_0 S^2}{2D}\left[1 + kt_1 + \frac{KT}{2}(m-1)\right] + K_1(\rho-1)^2 d(s)^{\alpha_1}$$

$$- \frac{i_e mC_0(TD-S)(D+S+MD-TD)}{D}\left\{1 + \frac{KT}{2}(m-1)\right\}$$

$$= mA_0 + mC_0 Q + \frac{mC_0 h(Q-S)^2}{2D}\left(1 - \frac{2Q(Q-S)}{3D}\right)$$

$$+ \frac{mC_0 S_0 S^2}{2D} - i_e mC_0(TD-S)\left(\frac{D+S+MD-TD}{D}\right)$$

$$+ mA_0\frac{KT}{2}(m-1) + mC_0 Q\frac{KT}{2}(m-1)$$

$$+\frac{mC_0h(Q-S)^2}{2D}\left(1-\frac{2\theta(Q-S)}{3D}\right)\frac{KT}{2}(m-1)$$

$$+\frac{mC_0S_0S^2}{2D}\left[Kt_1+\frac{KT}{2}(m-1)\right]+K_1(\rho-1)^2\,d(s)^{\alpha_1}$$

$$= mA_0+mC_0Q+\frac{mC_0h(Q-S)^2}{2D}+\frac{mC_0S_0S^2}{2D}$$

$$-i_e\frac{mC_0(TD-S)(D+S+MD-TD)}{D}$$

$$-\frac{mC_0h(Q-S)^2}{2D}\times 2\theta\frac{(Q-S)}{3D}$$

$$+mA_0\frac{KT}{2}(m-1)+mC_0Q\frac{KT}{2}(m-1)+$$

$$\frac{mC_0h(Q-S)^2}{2D}\times\frac{KT}{2}(m-1)$$

$$+\frac{mC_0S_0S^2}{2D}\left[K.\frac{TD-S}{D}+\frac{KT}{2}(m-1)\right]-i_emC_0\times$$

$$\frac{(TD-S)(D+S+MD-TD)}{D}\times\frac{KT}{2}(m-1)$$

$$-\frac{mC_0h(Q-S)^2}{2D}\times\frac{2\theta(Q-S)}{3D}\times\frac{KT}{2}(m-1)$$

$$+K_1(\rho-1)^2\,d(s)^{\alpha_1}$$

$$= m\left[\begin{array}{l} A_0+C_0\left\{\begin{array}{l} Q+\frac{h(Q-S)^2}{2D}+\frac{S_0S^2}{2D}\\ -i_e\frac{(TD-S)(D+S+MD-TD)}{D}\end{array}\right\}\\ -\frac{C_0h\begin{pmatrix}Q\\-S\end{pmatrix}^3\theta}{3D^2}+K\\ \left[\frac{(m-1)T}{2}\left\{\begin{array}{l} A_0+C_0(Q+\frac{h(Q-S)^2}{2D}+\frac{S_0S^2}{2D}\\ -\frac{S_0S^2(TD-S)}{D\times D(m-1)T}\\ -i_e\frac{TD-S}{D}\times(D+S+MD-TD))\\ -\frac{C_0h(Q-S)^3}{3D^2}\times\frac{T(m-1)}{2}\theta K\end{array}\right\}\right]\end{array}\right]$$

$$\therefore TC_3(QT)=\left[\begin{array}{l}F_{31}(Q,S)-\theta F_{32}(Q,S)\\ +K\,F_{33}(Q,S)-\theta K\,F_{34}(Q,S)\end{array}\right]$$

$$+K_1(\rho-1)^2\,d(s)^{\alpha_1}$$

Where $F_{31}=\left[A_0+C_0\left\{\begin{array}{l}Q+\frac{h(Q-S)^2}{2D}+\frac{S_0S^2}{2D}\\ -i_e(TD-S)\\ \times\frac{(D+S+MD-TD)}{D}\end{array}\right\}\right]$

$$F_{32}(Q,S)=\frac{C_0h(Q-S)^3}{3D^2}$$

$$F_{33}(Q,S)=\frac{(m-1)T}{2}\left\{\begin{array}{l}A_0\\ +C_0\left(\begin{array}{l}Q+\frac{h(Q-S)^2}{2D}\\ +\frac{S_0S^2}{2D}+\frac{S_0S^2(TD-S)}{D^2(m-1)T}\\ -i_e\frac{\mathrm{TD}-S}{D}\begin{pmatrix}D+S\\+MD-TD\end{pmatrix}\end{array}\right)\end{array}\right\}$$

$$=\frac{(m-1)T}{2}\left\{\begin{array}{l}\frac{C_0S_0S^2(TD-S)}{D^2(m-1)T}\\ +F_{31}(Q,S)\end{array}\right\}$$

$$\& F_{34}(Q,S)=\frac{(m-1)T\,C_0h(Q-S)^3}{2\times 3\times D^2}$$

$$=\frac{T(m-1)F_{32}(Q,S)}{2}$$

$$C_{pe}=K_1(\rho-1)^2\,d(S)^{\alpha_1}$$

4. Solution Procedure

Choosing two initial values of m^* say m and (m–1) in Z(m). Compute $T^*= Hm^*$ and then optimize Z(m) and Z(m–1) with the help of LINGO software to find the parameters Q^*, S^*, t_1^* in each case.

Step-1:

If $Z^*(m) \geq Z_i*(K+1)$ then set $m^* = k$ and stop.

5. Numerical Example

Here we have consider following numerical example.

Let A_0 = \$ 7.5 ,C_0= \$2.5, M = 0.3yr (case 1 only)
I_C = \$0.11, i_e = \$0.09, D = 500 units per year.
h=\$0.18, H = 1 year, K = 0.5, Q = 0.004
S_0 = \$0.5, k_1 = 2.00, $\alpha_1 = 1.0$

For the given data the total net value profit for the planning horizon π is Rs.74703.13, the number of replenishment during planning horizon *N*, is 26, per unit selling price of the items is Rs. 18.52, time with positive inventory T_1, is 0.2649512, replenishment cycle *T* is 0.3856304, selling price dependent declining quadratic demand *d(s)* is 570.3798 and the replenishment size *Q* is 220.0833. The total number of order is therefore *N*+1=27. All the decision parameters are compared with the other model related to the declined demand *d(S)* which is also related linearly to the selling price. It is observed that demand rate *d(s)*, order quantity *Q,* time with positive inventory T_1 and replenishment cycle T are more than the compared model. But only the number of replenishment during planning horizon, *N*, selling price S, unit selling prices and the total present value profit π are less from the compared model.

6. Sensitivity Analysis

It is interesting to investigate the influence of major parameters $A_0, C_0, M_1, I_c, I_e, D, h, H, K, \theta, S_0, S, m, T, t_1, Q, \rho, PE$ $and\ TC$ by taking six cases such that A_1, A_2, A_3, B_1, B_2 and B_3 are represented in appendix- B.

Case- A_1:

- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to the parameter A_0.
- S, m, T, t_1, Q, ρ $and\ PE$ are insensitive to the parameter C_0 but TC is moderately sensitive to C_0.
- S, m, T, t_1, ρ $and\ PE$ are insensitive to M_1, Q is moderately sensitive to M_1 but TC is highly sensitive to M_1.
- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to the parameter I_c.
- S, m, T, t_1, Q, ρ $and\ PE$ are insensitive to the parameter to I_e but TC is moderately sensitive to I_e.
- S, m, T, t_1, ρ $and\ PE$ are insensitive to D. Q and TC are moderately sensitive to D.
- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to h.
- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to parameter H.
- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to parameter K.
- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to parameter θ.
- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to parameter S_0.

Case- A_2:

- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to the parameter A_0.
- S and Q are moderately sensitive to the parameter C_0.
- C_0. m, T, t_1, ρ, PE $and\ TC$ are insensitive to the parameter C_0.
- T, t_1, ρ $and\ PE$ are insensitive to M_1. TC is moderately sensitive to M_1 but S and Q are highly sensitive to M_1.
- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to the parameter I_c.
- m, T, t_1, ρ $and\ PE$ are insensitive to the parameter to I_e but S and Q are highly sensitive to the parameter I_e. TC is moderately sensitive to I_e.
- S, m, T, t_1, Q, ρ $and\ PE$ are insensitive to D. TC are moderately sensitive to D.
- m, T, t_1, Q, ρ $and\ PE$ are insensitive to h. S and Q are highly sensitive to h and TC is moderately sensitive to h.
- m, T, t_1, ρ $and\ PE$ are insensitive to parameter H and S, Q and TC are moderately sensitive to H.
- m, T, t_1, ρ $and\ PE$ are insensitive to parameter K but S, Q and TC are moderately sensitive to the parameter K.
- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to parameter θ.
- m, T, t_1, ρ, PE $and\ TC$ are insensitive to parameter S_0 but S is highly sensitive to S_0. Q is moderately sensitive to S_0.
- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to the parameter K_1.
- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to the parameter α_1.

Case- A_3

- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to the parameter A_0.
- S and TC are moderately sensitive to C_0. m, T, t_1, Q, ρ $and\ PE$ are insensitive to the parameter C_0.
- m, T, t_1, ρ $and\ PE$ are insensitive to M_1. S, Q and TC are moderately sensitive to M_1.
- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to the parameter I_c.
- m, T, t_1, ρ $and\ PE$ are insensitive to the parameter to I_e but S is highly sensitive to the parameter I_e. Q and TC is moderately sensitive to I_e.
- S, m, T, t_1, ρ $and\ PE$ are insensitive to D. Q and TC are moderately sensitive to D.
- m, T, t_1, Q, ρ $and\ PE$ are insensitive to h. S, Q and T are moderately sensitive to h.
- m, T, t_1, ρ $and\ PE$ are insensitive to parameter H and S, Q and TC are moderately sensitive to H.
- m, T, t_1, ρ $and\ PE$ are insensitive to parameter K but S, Q and TC are moderately sensitive to the parameter K.
- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to parameter θ.
- m, T, t_1, ρ, PE $and\ TC$ are insensitive to parameter S_0 but S and Q is moderately sensitive to S_0.
- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to the parameter K_1.
- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to the parameter α_1.

Case- B_1

- $S, m, T, t_1, Q, \rho, PE$ $and\ TC$ are insensitive to the parameter A_0.

- $S, m, T, t_1, Q, \rho, PE$ and TC are insensitive to the parameter C_0.
- S, m, T, t_1, ρ and PE are insensitive to M_1. Q and TC are moderately sensitive to M_1.
- $S, m, T, t_1, Q, \rho, PE$ and TC are insensitive to the parameter I_c.
- S, m, T, t_1, Q, ρ and PE are insensitive to the parameter to I_e but TC is moderately sensitive to I_e.
- $S, m, T, t_1,$ and ρ are insensitive to D. Q, PE and TC are moderately sensitive to D.
- S, m, T, t_1, Q, ρ and PE are insensitive to h. TC is moderately sensitive to h.
- m, T, t_1, ρ and PE are insensitive to parameter H and S, Q and TC are moderately sensitive to H.
- m, T, t_1, ρ and PE are insensitive to parameter K but S, Q and TC are moderately sensitive to the parameter K.
- $S, m, T, t_1, Q, \rho, PE$ and TC are insensitive to parameter θ.
- m, T, t_1, ρ, PE and TC are insensitive to parameter S_0 but S and Q is moderately sensitive to S_0. ...
- $S, m, T, t_1, Q, \rho, PE$ and TC are insensitive to the parameter K_1.
- $S, m, T, t_1, Q, \rho, PE$ and TC are insensitive to the parameter α_1.

Case-B2= Case-B3

- $S, m, T, t_1, Q, \rho, PE$ and TC are insensitive to the parameter A_0.
- S and TC are moderately sensitive to C_0. m, T, t_1, Q, ρ and PE are insensitive to the parameter C_0.
- m, T, t_1, ρ and PE are insensitive to M_1. S, Q and TC are moderately sensitive to M_1.
- $S, m, T, t_1, Q, \rho, PE$ and TC are insensitive to the parameter I_c.
- m, T, t_1, ρ and PE are insensitive to the parameter to I_e but S is highly sensitive to the parameter I_e. Q and TC is moderately sensitive to I_e.
- S, m, T, t_1, ρ and PE are insensitive to D. Q and TC are moderately sensitive to D.
- m, T, t_1, Q, ρ and PE are insensitive to h. S, Q and T are moderately sensitive to h.
- m, T, t_1, ρ and PE are insensitive to parameter H and S, Q and TC are moderately sensitive to H.
- m, T, t_1, ρ and PE are insensitive to parameter K but S, Q and TC are moderately sensitive to the parameter K.
- $S, m, T, t_1, Q, \rho, PE$ and TC are insensitive to parameter θ.
- m, T, t_1, ρ, PE and TC are insensitive to parameter S_0 but S and Q is moderately sensitive to S_0.
- $S, m, T, t_1, Q, \rho, PE$ and TC are insensitive to the parameter K_1.
- $S, m, T, t_1, Q, \rho, PE$ and TC are insensitive to the parameter α_1.

7. Conclusion

In this paper, an EPQ model is introduced which investigates the optimal replenishment quantity. Unit selling price, replenishment cycle, time with positive invention the total value net profit with finite planning horizon for deteriorating items. The model considers the impact of price dependant quadratic demand, and shortages and varying rate of deterioration. The model can be used for electronics and other luxury products which are more likely to have the above characteristics. This paper provides a useful property for finding the optimal net present value profit with finite planning horizon for deteriorating items. A new mathematical model with decline quadratic demand is developed and compared to the other EPQ model with decline linear demand and numerically. The economic replenishment quantity Q^* and net present value profit π^* for the present model were found to be more than that of the compared model respectively. Hence the utilization of selling price dependent declined quadratic demand makes the scope of application broader. Lingo 13.0 version software is used to derive the optimal number of replenishment and unit price to maximize the total present value net profit. Further, a numerical example is presented to illustrate the theoretical results, and some observations are obtained from sensitivity analysis with respect to the major parameters controlling the market demand through the manipulation of selling price is an important strategy for increasing profit. This can be achieved by using the joint optimal replenishment and pricing strategy developed in this study. In the future study, it is hoped further incorporate the proposed model into several situations such as stochastic market demand, fuzzy decision parameters and partial back logging.

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Appendix -A

$$Q-S=I_1(0)=\frac{D}{\theta}\left(e^{\theta t_1}-1\right)............A1$$

$$=\frac{D}{\theta}\left[1+\theta t_1+\frac{\theta^2t_1^2}{2}+\frac{\theta^3t_1^3}{6}+\frac{\theta^4t_1^4}{24}+.................1\right]$$

$$=D\left[t_1+\frac{\theta t_1^2}{2}+\frac{\theta^2t_1^3}{6}+\frac{\theta^3t_1^4}{24}+.................\right]$$

$$=D\left[\left(t_1+\frac{\theta t_1^2}{2}\right)+\theta^2\left(\frac{t_1^3}{6}+\frac{\theta t_1^4}{24}+.................\right)\right]$$

$$\approx D\left(t_1+\frac{\theta t_1^2}{2}\right)\left(Since\,\theta\le 1 So, \theta^2 \,\&\, higher\ \ power\, of\ \ \theta\ \ neglected\right)$$

From equation A_1 we have $e^{\theta t_1}=1+\frac{\theta(Q-S)}{D}$

$$Q-S=\frac{D}{\theta}\left(e^{\theta t_1}-1\right)$$

$$\Rightarrow\qquad e\theta t_1-1=\frac{(Q-S)\theta}{D}$$

$$\Rightarrow\qquad e^{\theta t_1}=1+\frac{(Q-S)\theta}{D}$$

$$\Rightarrow\qquad \theta t_1=\log\left[1+\frac{(Q-S)\theta}{D}\right]$$

$$\Rightarrow\qquad t_1=\frac{1}{\theta}\log\left[1+\frac{(Q-S)\theta}{D}\right]=\frac{1}{\theta}\left[\frac{\theta(Q-S)}{D}-\frac{\theta^2(Q-S)^2}{2D^2}+\frac{\theta^3(Q-S)^3}{3D^3}.......\right]=$$

$$\left[\frac{Q-S}{D}.\frac{\theta(Q-S)^2}{2D^2}+\frac{\theta^2(Q-S)^3}{3D^3}........\right]$$

Since $\theta\le 1 So\ neglect\,\theta^2$ ....$A3$

$$t_1^2=\frac{(Q-S)^2}{D^2}-\frac{\theta(Q-S)^3}{D^3}$$

$$t_1^3=\left[\frac{Q-S}{D}-\frac{\theta(Q-S)^2}{2D^2}\right]^3$$

$$=\left(\frac{Q-S}{D}\right)^3-\frac{\theta^3(Q-S)^6}{8D^6}-3\frac{(Q-S)^2}{D^2}\frac{\theta(Q-S)^2}{2D^2}+3\left(\frac{Q-S}{D}\right).\frac{\theta^2(Q-S)^4}{4D^4}$$

$$\Rightarrow\frac{\theta t_1^3}{6}=\frac{\theta(Q-S)^3}{6D^3}-\frac{\theta^4(Q-S)^6}{48D^6}-\frac{3\theta^2(Q-S)^2}{12D^4}+3\frac{\theta^3(Q-S)^5}{24D^5}$$

$$=\frac{\theta(Q-S)^3}{6D^3}-\frac{\theta^4(Q-S)^6}{48D^6}-\frac{\theta^2(Q-S)^4}{4D^4}+\frac{\theta^3(Q-S)^6}{8D^5}$$

$$\Rightarrow\qquad \frac{\theta t_1^3}{6}\approx\frac{\theta(Q-S)^3}{QD^3}$$ as $\theta\le 1\ neglecting\ \ higher\ power\, of\ \ \theta$

The holding cost (O, H)

$$C_h = h\sum_{n=0}^{n-1} C(nT)\int_0^{t_1} I_1(t)\,dt = h\sum_{n=0}^{n-1} C(nt)\int_0^{t_1}\frac{D}{\theta}\left(e^{\theta(t_1-t)}-1\right)dt$$

$$= h\sum_{n=0}^{m-1} C(nT)\left\{\frac{D}{\theta}\int_0^{t_1}\left(e^{\theta t_1}.e^{-\theta t}-1\right)dt\right\}$$

$$= h\sum_{n=0}^{m-1} C(nT)\left\{\frac{D}{\theta}\left[\frac{e^{\theta t_1}e^{-\theta t}}{\theta}-t\right]_0^{t_1}\right\}$$

$$= h\sum_{n=0}^{m-1} C(nT)\left\{\frac{D}{\theta^2}\left[-e^{\theta(t_1-t)}-t\theta\right]_0^{t_1}\right\}$$

$$= h\sum_{n=0}^{m-1} C(nT)\left\{\frac{D}{\theta^2}\left(e^{\theta t_1}-1-t_1\theta\right)\right\}$$

$$= \frac{hD}{\theta^2}\left(e^{\theta t_1}-t_1\theta-1\right)\sum_{n=0}^{m-1} C(nT)$$

$$= \frac{hD}{\theta^2}\left(e^{\theta t_1}-t_1\theta-1\right)\sigma\sum_{n=0}^{m-1}(1+knt)$$

$$= \frac{hD}{\theta^2}\left(e^{\theta t_1}-t_1\theta-1\right)C_0\left[m+\frac{KT(m)(m-1)}{2}\right]$$

$$= \frac{C_0hD}{\theta^2}\left[e^{\theta t_1}-t_1\theta-1\right]m\left[1+\frac{KT(m-1)}{2}\right]$$

$$= \frac{mC_0hD}{\theta^2}\left[1+\theta t_1+\frac{\theta^2t_1^2}{2}+\frac{\theta^3t_1^3}{6}+........-\theta t_1-1\right]\left[1+\frac{KT(m-1)}{2}\right]$$

$$= \frac{mC_0hD}{\theta^2}\left[\frac{\theta^2t_1^2}{2}+\frac{\theta^3t_1^3}{6}+........\right]\left[1+\frac{KT(m-1)}{2}\right]$$

$$= mC_0hD\left[\frac{t_1^2}{2}+\frac{\theta t_1^3}{6}\right]\left[1+\frac{KT(m-1)}{2}\right]$$

$$= mC_0hD\left[\frac{t_1^2}{2}+\frac{\theta t_1^3}{6}\right]\left[1+\frac{KT(m-1)}{2}\right]$$

Now $\frac{t_1^2}{2}+\frac{\theta t_1^3}{6}=\frac{1}{2}\left[\left[\frac{Q-S}{D}-\frac{\theta(Q-S)^2}{2D^2}\right]^2\right]+\frac{\theta(Q-S)^3}{6D^3}$

$$\frac{1}{2}\frac{\theta(Q-S)^2}{D^2}+\frac{1}{2}\frac{\theta^2(Q-S)^4}{4D^2}-\frac{\theta(Q-S)^3}{2D^3}+\frac{\theta(Q-S)^3}{6D^3}$$

$$\approx\frac{(Q-S)^2}{2D^2}-\frac{1}{3}\frac{\theta(Q-S)^3}{D^3}$$

$$\Rightarrow\frac{t_1^2}{2}+\frac{\theta t_1^3}{6}\approx\frac{(Q-S)^2}{2D^2}-\frac{\theta(Q-S)^3}{3D^3}$$

Hence $C_h = mC_0hD\left[\frac{(Q-S)}{2D^2}-\frac{\theta(Q-S)^3}{3D^3}\right]\left[1+\frac{KT(m-1)}{2}\right]$

$$\frac{mC_0hD(Q-S)^2}{2D^2}\left[1-\frac{2\theta(Q-S)}{3D}\right]\left[1+\frac{KT(m-1)}{2}\right]$$

$$C_h=\frac{mC_0h(Q-S)^2}{2D}\left[1-\frac{2\theta(Q-S)}{3D}\right]\left[1+\frac{KT(m-1)}{2}\right]$$

$$C_i = i_c \sum_{n=0}^{m-1} C(nT) \int_{\mu}^{t_1} I_1(t)\,dt$$

$$= i_c \sum_{n=0}^{m-1} C_0(1+KnT) \int \frac{D}{\theta}\left[e^{\theta(t_1-t)} - 1\right]dt$$

$$= \frac{i_c C_0 D}{\theta}\left[m + \frac{KT(m-1)(m)}{2}\right]\left[-\frac{1}{\theta} - t_1 + \frac{e^{\theta(t_1-t)^4}}{\theta} + M\right]$$

$$= \frac{i_c C_0 Dm}{\theta^2}\left[m + \frac{KT(m-1)(m)}{2}\right]\left[\theta M + e^{\theta(t_1-M)} - \theta t_1 - 1\right]$$

$$= \frac{i_c C_0 Dm}{\theta^2}\left[m + \frac{KT(m-1)(m)}{2}\right]\left[1 + \theta(t_1 - M) + \frac{\theta^2(t_1-M)^2}{2} + ... + \theta M - \theta t_1 - 1\right]$$

$$= \frac{i_c C_0 D}{\theta^2}\left[\frac{\theta^2(t_1-M)^2}{2} + \frac{\theta^3(t_1-M)^3}{6} + ...\right]\left[1\frac{KT(m-1)}{2}\right]$$

$$= i_c C_0 DM\left[\frac{(t_1-M)^2}{2} + \frac{1}{6}\theta(t_1-M)^3\right]\left[1 + \frac{KT(m-1)}{2}\right] .....neglect\ \theta^2, \theta^3.....$$

$$C_i = i_e C_0 DM\left[\frac{1}{2}\left(t_1 - M^2\right) + \frac{1}{6}(t_1-M)^3\theta\right]\left[1 + \frac{KT}{2}(m-1)\right]$$

But we know that

$$t_1 = \frac{Q-S}{D} = \frac{\theta(Q-S)^2}{2D^2}$$

$$\Rightarrow (t_1 - M) = \frac{Q-S}{D} - \frac{\theta(Q-S)^2}{2D^2} - M$$

$$= \frac{Q-S-MD}{D} - \frac{\theta(Q-S)^2}{2D^2}$$

$$\Rightarrow (t_1-M)^2 = \left\{\frac{Q-S-MD}{D} - \frac{\theta(Q-S)^2}{2D^2}\right\}^2$$

$$\approx \left\{\frac{\left(Q-S-MD^2\right)}{D^2} - \frac{\theta(Q-S-MD)(Q-S)^2\theta}{D^3}\right\}$$

Neglect higher part of θ

$$(t_1-M)^3 = \left\{\frac{Q-S-MD}{D} - \frac{\theta(Q-S)^2}{2D^2}\right\}^3$$

$$\approx \frac{(Q-S-MD)^3}{D^3} - \frac{3\theta(Q-S-MD)^2(Q-S)^2}{2D^4}$$

Hence $\frac{1}{2}(t_1-M)^2 + \frac{\theta}{6}(t_1-M)^3$

$$\frac{1}{2}\frac{(Q-S-MD)^2}{D^2} - \frac{1}{2}\frac{(Q-S-MD)(Q-S)^2\theta}{D^3}$$

$$+ \frac{\theta}{6}\frac{(Q-S-MD)^3}{D^3} - \frac{\theta}{6}\frac{3\theta(Q-S-MD)^2(Q-S)^2}{2D^4}$$

$$= \frac{Q-S-MD}{2D}\left[\frac{(Q-S-MD)}{D} - \frac{(Q-S)^2\theta}{D^2} + \frac{\theta}{3}\frac{(Q-S-MD)^2}{D^2}\right]$$

$$=\frac{Q-S-MD}{2D}\left[\frac{Q-S}{D}-M-\theta\left\{\frac{(Q-S)^2}{D^2}-\frac{1}{3}\left(\frac{Q-S}{D}-M\right)^2\right\}\right]$$

Hence $C_i=mC_0i_ci_cD\left\{\frac{1}{2}(t_1-M)^2+\frac{\theta}{6}(t_1-M)^3\right\}\left\{1+\frac{KT}{2}(m-1)\right\}$

$$\Rightarrow C_i=MC_0i_cD\frac{(Q-S-MD)}{2D}\left[\frac{Q-S}{D}-M-\theta\left\{\frac{(Q-S)^2}{D^2}-\frac{1}{3}\left(\frac{Q-S}{D}-M\right)^2\right\}\right]\left[1+\frac{KT}{2}(m-1)\right]$$

$$\Rightarrow C_i=\frac{MC_0i_c\,(Q-S-MD)}{2}\left[\frac{Q-S}{D}-M-\theta\left\{\frac{2}{3}\frac{(Q-S)^2}{D^2}\right\}+\frac{2}{3}\frac{m(Q-S)}{D}-\frac{M^2}{3}\right]\left[1+\frac{KT}{2}(m-1)\right]$$

Hence $\Rightarrow C_i=\frac{i_cMC_0}{2}(Q-S-MD)\left[\frac{Q-S-MD}{D}-\theta\left\{\frac{2(Q-S)(Q-S+MD)}{3D^2}-\frac{M^2}{3}\right\}\right]\left[1+\frac{KT}{2}(m-1)\right]$

Appendix -B

Case A1

Parameter	Value	Iteration	S	m	T	t_1	Q	ρ	PE	TC	TC%
A_0	7.1	64	0	2	0.5	0.5	150.1804	1	0	807.202	.111314473
	7.4	66	0	2	0.5	0.5	150.1804	1	0	808.2952	.027828618
	7.6	64	0	2	0.5	0.5	150.1804	1	0	808.7452	.027
C_0	2.4	65	0	2	0.5	0.5	150.1804	1	0	776.8544	3.9165
	2.6	65	0	2	0.5	0.5	150.1804	1	0	840.1860	3.9165
	2.7	69	0	2	0.5	0.5	150.1804	1	0	871.8579	7.8330
M_1	0.20	60	0	2	0.5	0.5	100.0801	0.99999	0	539.3365	33.2933
	0.21	59	0	2	0.5	0.5	105.0883	1	0	566.0160	29.9935
	0.22	59	0	2	0.5	0.5	110.0970	1	0	592.7486	26.6872
I_C	0.1101	64	0	2	0.5	0.5	150.1804	1	0	808.5202	0
	0.1102	64	0	2	0.5	0.5	150.1804	1	0	808.5202	0
	0.1105	64	0	2	0.5	0.5	150.1804	1	0	808.5202	0
I_e	0.08	49	0	2	0.5	0.5	150.1804	1	0	816.9577	1.0435
	0.10	69	0	2	0.5	0.5	150.1804	1	0	800.0827	1.0435
	0.101	69	0	2	0.5	0.5	150.1804	1	0	799.2390	1.1479
D	480	64	0	2	0.5	0.5	144.1732	1	0	776.8544	3.9165
	481	64	0	2	0.5	0.5	144.4736	1	0	778.4377	3.7206
	482	64	0	2	0.5	0.5	144.7739	1	0	780.0210	3.5248
h	0.14	65	0	2	0.5	0.5	150.1804	1	0	803.4496	0.6271
	0.16	64	0	2	0.5	0.5	150.1804	1	0	805.9849	0.3135
	0.20	64	0	2	0.5	0.5	150.1804	1	0	811.0555	0.3135
H	1.1	66	0	2	0.5	0.5	150.1804	1	0	817.5038	1.1111
	1.101	66	0	2	0.5	0.5	150.1804	1	0	817.5936	1.1222
	1.102	66	0	2	0.5	0.5	150.1804	1	0	817.6535	1.1333
K	0.49	66	0	2	0.5	0.5	150.1804	1	0	806.7235	0.2222
	0.51	64	0	2	0.5	0.5	150.1804	1	0	810.3169	0.2222
	0.52	68	0	2	0.5	0.5	150.1804	1	0	812.1137	0.4444
θ	0.002	65	0	2	0.5	0.5	150.1804	1	0	807.9939	0.0650
	0.003	65	0	2	0.5	0.5	150.1804	1	0	808.2569	0.0325
	0.005	66	0	2	0.5	0.5	150.1804	1	0	808.7839	0.0326
S_0	0.2	64	0	2	0.5	0.5	150.1804	1	0	808.5202	0
	0.3	64	0	2	0.5	0.5	150.1804	1	0	808.5202	0
	0.4	64	0	2	0.5	0.5	150.1804	1	0	808.5202	0
K_1	0.5.1	64	0	2	0.5	0.5	150.1804	1	0	808.5202	
	0.5.0.5	60.5	0	2	0.5	0.5	150.1804	1	0	808.5202	
	0.5.4	70	0	2	0.5	0.5	150.1804	1	0	808.5202	
α_1	1.1	56	0	2	0.5	0.5	150.1804	1	0	808.5202	0
	1.2	56	0	2	0.5	0.5	150.1804	1	0	808.5202	0
	1.3	55	0	2	0.5	0.5	150.1804	1	0	808.5202	0

Case A2 $M=t_1$

Parameter	Value	Iteration	S	M	T	t_1	Q	ρ	PE	TC	TC%
A_0	7.1	101	64.56014	2.330349	0.4291203	0.3	214.7406	1	0	1401.334	0.0752
	7.4	98	67.63357	2.297440	0.4352671	0.3	217.8140	1	0	1402.127	0.0186
	7.6	67	69.69357	2.275897	0.4393871	0.3	219.8740	1	0	1402.649	0.0185
C_0	2.4	99	71.89285	2.253340	0.4437857	0.3	222.0733	1	0	1347.069	3.9446
	2.6	67	65.70061	2.318028	0.4314012	0.3	215.8810	1	0	1457.696	3.9437
	2.7	70	62.97371	2.347708	0.4259474	0.3	213.1541	1	0	1512.992	7.8867
M_1	0.21	82	1.376093	4.700304	0.2127522	0.21	106.4644	1	0	1434.503	2.2899
	0.22	86	7.979468	4.238026	0.2359589	0.22	118.0764	1	0	1430.872	2.0310
	0.23	126	14.80668	3.857882	0.22596134	0.23	129.9127	1	0	1427.274	1.7744
I_C	0.1101	67	68.66241	2.286630	0.4373248	0.3	218.8428	1	0	1402.389	0
	0.1102	67	68.60241	2.286630	0.4373248	0.3	218.8428	1	0	1402.389	0
	0.1105	67	68.66241	2.286630	0.4373248	0.3	218.8428	1	0	1402.389	0
I_e	0.10	72	31.03489	2.781898	0.3620698	0.3	181.2153	1	0	1391.557	0.7723
	0.101	77	27.31384	2.819859	0.3546277	0.3	177.4943	1	0	1390.342	0.8590
	0.102	84	23.58782	2.880386	0.3471756	0.3	173.7683	1	0	1389.100	0.9475
D	480	86	69.01714	2.253340	0.4437857	0.3	213.1904	1	0	1347.009	3.9489
	481	91	68.99882	2.255052	0.4434487	0.3	213.4724	1	0	1347.836	3.7473
	482	103	68.98058	2.256760	0.4431132	0.3	213.7545	1	0	1352.602	3.5501
h	0.10	63	23.51602	2.881578	0.3470320	0.3	173.6964	1	0	1389.076	0.9493
	0.11	63	29.11212	2.791548	0.3582242	0.3	179.2925	1	0	1390.933	0.8168
	0.12	63	34.70035	2.707087	0.3644007	0.3	184.8808	1	0	1392.728	0.6888
H	1.1	68	59.95580	2.619599	0.4199116	0.3	210.1362	1	0	1576.328	12.4030
	1.101	91	59.87780	2.622955	0.4197556	0.3	210.0582	1	0	1578.097	12.5291
	1.102	91	59.87780	2.622955	0.4197556	0.3	210.0582	1	0	1578.097	12.5291
K	0.49	66	61.41694	2.364995	0.4228339	0.3	211.5974	1	0	1398..858	0.2517
	0.51	65	77.15393	2.201151	0.4543079	0.3	227.3344	1	0	1405.842	0.2462
	0.52	94	87.51674	2.105115	0.4750335	0.3	237.6972	1	0	1409.203	0.4858
θ	0.002	65	66.25897	2.312043	0.4325179	0.3	216.3491	1	0	1401.775	.0437
	0.003	65	67.45845	2.299290	0.4349169	0.3	217.5937	1	0	1402.082	.0218
	0.005	63	69.87101	2.274061	0.4397420	0.3	220.0967	1	0	1402.693	.0216
S_0	0.6	60	36.57086	2.679947	0.3731417	0.3	186.7513	1	0	1404.381	0.1420
	0.7	63	25.40634	2.850524	0.3508127	0.3	175.5868	1	0	1405.220	0.2018
	0.8	63	19.53628	2.949221	0.3390726	0.3	169.7167	1	0	1405.693	0.2355
K_1	2.1	67	68.66241	2.286630	0.4373248	0.3	218.8428	1	0	1402.389	0
	2.2	67	68.66241	2.286630	0.4373248	0.3	218.8428	1	0	1402.384	0
	2.4	67	68.66241	2.286630	0.4373248	0.3	218.8428	1	0	1402.389	0
α_1	1.1	66	68.66241	2.286630	0.4373248	0.3	218.8428	1	0	1402.389	0
	1.2	68	68.66241	2.286630	0.4373248	0.3	218.8428	1	0	1402.389	0
	1.3	77	68.66241	2.286630	0.4373248	0.3	218.8428	1	0	1402.389	0

Case A3 $t_1<M_1<T$

Parameter	Value	Iteration	S	M	T	t_1	Q	ρ	PE	TC	TC%
A_0	7.1	74	64.56013	2.330349	0.4291203	0.3	214.7406	1	0	1401.334	0.07522
	7.4	66	67.63357	2.297440	0.4352671	0.3	217.8140	1	0	1402.127	0.0186
	7.6	68	69.69357	2.275897	0.4352671	0.3	219.8740	1	0	1402.649	0.0185
C_0	2.4	68	71.89285	2.253340	0.4437857	0.3	222.0733	1	0	1374.069	3.9446
	2.6	69	65.70061	2.318028	0.4314012	0.3	215.8810	1	0	1457.696	3.9437
	2.7	74	62.97371	2.347708	0.4259474	0.3	213.1541	1	0	1512.992	7.8867
M_1	0.31	91	77.37351	2.151708	0.464770	0.3	232.5661	1	0	1398.828	0.2539
	0.311	71	78.26252	2.138923	0.4675250	0.3	233.9564	1	0	1398.472	0.2793
	0.32	70	86.42348	2.029027	0.4928470	0.3	246.6288	1	0	1395.259	0.5084
I_C	0.1101	69	68.66241	2.286630	0.4373248	0.3	218.8425	1	0	1402.389	0
	0.1102	64	68.66241	2.286630	0.4373248	0.3	218.8425	1	0	1402.389	0
	0.1103	69	68.66241	2.286630	0.4373248	0.3	218.8425	1	0	1402.389	0
I_e	0.10	67	31.03489	2.761898	0.3620698	0.3	181.2153	1	0	1391.557	0.7723
	0.101	62	27.31384	2.819859	0.3546277	0.3	177.4943	1	0	1390.342	0.8590
	0.102	70	23.58782	2.880386	0.3471756	0.3	173.7683	1	0	1398.100	0.9475
D	480	75	69.01714	2.253340	0.4437857	0.3	213.1904	1	0	1347.069	3.9489
	481	66	68.99882	2.255052	0.4434487	0.3	213.4724	1	0	1349.836	3.7473
	482	80	68.98058	2.256760	0.4431132	0.3	213.7545	1	0	1352.602	3.5501

h	0.14	61	45.89491	2.552389	0.3917898	0.3	196.0753	1	0	1396.148	0.4450
	0.16	60	57.18474	20413305	0.4143695	0.3	207.3652	1	0	1399.362	0.2158
	0.19	67	74.50468	2.227125	0.4490094	0.3	224.6851	1	0	1403.837	0.1032
H	1.1	69	59.95580	2.619599	0.4149116	0.3	210.1362	1	0	1576.328	12.4030
	1.101	69	59.87780	2.622955	0.4197556	0.3	210.0582	1	0	1578.097	12.5291
	1.102	72	59.79995	2.626311	0.4195999	0.3	209.9804	1	0	1579.868	12.6554
K	0.49	74	61.41694	2.364995	0.4228339	0.3	211.5974	1	0	1398.858	0.2517
	0.51	68	77.15393	2.201151	0.4543079	0.3	227.3344	1	0	1405.842	0.2462
	0.52	68	87.51674	2.105115	0.4750335	0.3	237.6972	1	0	1409.203	0.4858
θ	0.002	68	66.25897	2.312043	0.4325179	0.3	216.3491	1	0	1401.775	0.0437
	0.003	68	67.45845	2.299240	0.4349169	0.3	217.5937	1	0	1402.082	0.0218
	0.005	68	69.87101	2.274061	0.4397420	0.3	220.0967	1	0	1402.693	0.0216
S_0	0.51	68	62.7222	2.350483	0.4254444	0.3	212.9027	1	0	1402.711	0.0229
	0.53	67	53.78680	2.453545	0.4075736	0.3	203.9672	1	0	1403.235	0.0603
	0.54	67	50.30232	2.496277	0.4006046	0.3	200.4828	1	0	1403.453	0.0758
K_1	2.1	66	68.66241	2.286630	0.4373248	0.3	218.8428	1	0	1402.389	0
	2.2	68	68.66241	2.286630	0.4373248	0.3	218.8428	1	0	1402.389	0
	2.4	67	68.66241	2.286630	0.4373248	0.3	218.8428	1	0	1402.389	0
α_1	1.1	65	68.66241	2.286630	0.4373248	0.3	218.8428	1	0	1402.389	0
	1.11	80	68.66241	2.286630	0.4373248	0.3	218.8428	1	0	1402.389	0
	1.12	64	68.66241	2.286630	0.4373248	0.3	218.8428	1	0	1402.389	0

Case B1 $M_1 < t_1$

Parameter	Value	Iteration	S	m	T	t_1	Q	ρ	PE	TC	TC%
A_0	7.1	21	0	2	0.5	0.5	150.1804	3	4000	4807.620	0.018716777
	7.4	21	0	2	0.5	0.5	150.1804	3	4000	4808.295	4.67919443
	7.6	21	0	2	0.5	0.5	150.1804	3	4000	4808.745	4.67919443
C_0	2.4	21	0	2	0.5	0.5	150.1804	3	4000	4776.854	1.369526228
	2.6	21	0	2	0.5	0.5	150.1804	3	4000	4840.186	0.658539425
	2.7	21	0	2	0.5	0.5	150.1804	3	4000	4871.852	1.317078852
M_1	0.21	24	0	2	0.5	0.5	105.0883	3	4000	4568.016	5.04321496
	0.22	24	0	2	0.5	0.5	110.0973	3	4000	4592.749	0.044872642
	0.23	24	0	2	0.5	0.5	115.1060	3	4000	4619.534	3.930232171
I_C	0.1101	21	0	2	0.5	0.5	150.1804	3	4000	4808.520	0
	0.1102	21	0	2	0.5	0.5	150.1804	3	4000	4808.520	0
	0.1105	21	0	2	0.5	0.5	150.1804	3	4000	4808.520	0
I_e	0.100	21	0	2	0.5	0.5	150.1804	3	4000	4800.083	0.175459392
	0.101	21	0	2	0.5	0.5	150.1804	3	4000	4799.239	0.193011571
	0.102	21	0	2	0.5	0.5	150.1804	3	4000	4798.395	0.210563749
D	480	21	0	2	0.5	0.5	144.1732	3	3840	4616.854	3.985966576
	481	21	0	2	0.5	0.5	144.4736	3	3848	4626.438	3.78665369
	482	21	0	2	0.5	0.5	144.7739	3	3856	4636.021	3.5873616
h	0.10	22	0	2	0.5	0.5	150.1804	3	4000	4798.379	0.210896492
	0.11	22	0	2	0.5	0.5	150.1804	3	4000	4799.647	0.184526631
	0.12	21	0	2	0.5	0.5	150.1804	3	4000	4800.914	0.158177568
H	1.1	21	0	2	0.55	0.55	150.1804	3	4000	4817.504	0.186835034
	1.101	21	0	2	0.5505	0.5505	150.1804	3	4000	4517.594	0.188706712
	0.102	21	0	2	0.5	0.5510	150.1804	3	4000	4817.683	0.190557593
K	0.49	21	0	2	0.5	0.5	150.1804	3	4000	4806.724	0.037350369
	0.51	21	0	2	0.5	0.5	150.1804	3	4000	4810.317	0.037371166
	0.52	21	0	2	0.5	0.5	150.1804	3	4000	4812.114	0.074742332
θ	0.002	21	0	2	0.5	0.5	150.0901	3	4000	4807.994	0.010938916
	0.003	21	0	2	0.5	0.5	150.1352	3	4000	4808.257	5.469458378
	0.005	21	0	2	0.5	0.5	150.2257	3	4000	4808.784	5.490254798
S_0	0.6	21	0	2	0.5	0.5	150.1804	3	4000	4808.520	0
	0.7	21	0	2	0.5	0.5	150.1804	3	4000	4808.520	0
	0.8	21	0	2	0.5	0.5	150.1804	3	4000	4808.520	0
K_1	2.1	21	0	2	0.5	0.5	150.1804	3	4200	4808.520	4.159283938
	2.2	21	0	2	0.5	0.5	150.1804	3	4400	4808.520	8.318567875
	2.4	21	0	2	0.5	0.5	150.1804	3	4800	4808.520	16.63713575
α_1	1.1	21	0	2	0.5	0.5	150.1804	3	7446.582	8255.103	7.167658656
	1.2	21	0	2	0.5	0.5	150.1804	3	13862.900	14671.42	205.1130077
	1.3	21	0	2	0.5	0.5	150.1804	3	25807.800	2616.32	45.58991124

Case B2 = Case B3

Parameter	Value	Iteration	S	m	T	t_1	Q	ρ	PE	TC	TC%
A_0	7.1	85	64.56014	2.330349	0.4291203	0.3	214.7406	2	1000	2401.334	0.043914619
	7.4	84	67.63357	2.297440	0.4352671	0.3	217.8140	2	1000	2402.127	0.01090581
	7.6	93	69.69357	2.275897	0.4393871	0.3	219.8740	2	1000	2402.649	0.01082256
C_0	2.4	66	71.89285	2.253340	0.4437857	0.3	222.0733	2	1000	2347.069	2.302707846
	2.6	111	65.70061	2.318028	0.4314012	0.3	215.8810	2	1000	2457.696	2.302166718
	2.7	55	62.93371	2.347708	0.4259473	0.3	213.1541	2	1000	2512.992	4.603875559
M_1	0.27	60	44.21128	2.790003	0.2584226	0.27	179.3574	2	1000	2413.038	0.141789445
	0.28	80	52.10818	2.602700	0.3842764	0.28	192.2653	2	1000	2409.491	0.295622399
	0.29	90	60.25077	2.436044	0.4105015	0.29	205.4194	2	1000	2405.942	0.443267097
I_C	0.1101	103	68.66241	2.286630	0.4373248	0.3	218.8428	2	1000	2402.389	0
	0.1102	85	68.66241	2.286630	0.4373248	0.3	218.8428	2	1000	2402.389	0
	0.1105	74	68.66241	2.286630	0.4373248	0.3	218.8428	2	1000	2402.389	0
I_e	0.100	73	31.034489	2.761898	0.3620698	0.3	181.2153	2	1000	2391.557	0.450884515
	0.101	66	27.31384	2.819859	0.3546277	0.3	177.4943	2	1000	2390.342	0.501459172
	0.103	71	19.85370	2.943710	0.3397074	0.3	170.0341	2	1000	2387.829	0.606063381
D	480	63	69.01714	2.253340	0.4437857	0.3	213.1904	2	960	2307.069	3.967717135
	481	69	68.99882	2.255052	0.4434487	0.3	231.4724	2	962	2311.836	3.769289653
	482	65	68.98058	2.256760	0.4431132	0.3	213.4745	2	964	2316.602	3.570903796
h	0.10	65	23.51602	2.881578	0.3470320	0.3	173.6964	2	1000	2389.076	0.554156716
	0.11	74	29.11212	2.791548	0.3582242	0.3	179.2925	2	1000	2390.933	0.47685866
	0.12	64	34.70035	2.707087	0.3694007	0.3	184.8808	2	1000	2392.728	0.402141368
H	1.1	83	59.95580	2.619599	0.4199116	0.3	210.1362	2	1000	2576.320	7.239918265
	1.101	98	59.87780	2.622955	0.4197556	0.3	210.0582	2	1000	2578.097	7.313844677
	0.102	86	59.79995	2.626311	0.4195999	0.3	209.9804	2	1000	2579.868	7.387604589
K	0.49	75	61.41694	2.364995	0.4228339	0.3	211.5974	2	1000	2398.585	0.146978695
	0.51	70	77.15393	2.201151	0.4543079	0.3	227.3344	2	1000	2405.842	0.143731926
	0.52	89	87.51674	2.105115	0.4750335	0.3	237.6972	2	1000	2409.203	0.283634332
θ	0.002	92	66.25892	2.312043	0.43215179	0.3	216.3491	2	1000	2401.775	0.025557892
	0.003	76	67.45845	2.299290	0.4349169	0.3	217.5937	2	1000	2402.082	0.012778946
	0.005	80	69.87101	2.274061	0.4397420	0.3	220.0967	2	1000	2402.693	0.01265407
S_0	0.6	85	36.57086	2.679947	0.3731417	0.3	186.7513	2	1000	2404.381	0.082917462
	0.7	77	25.40634	2.850524	0.3508127	0.3	175.5868	2	1000	2405.220	0.117841032
	0.8	82	19.53628	2.949221	0.3390726	0.3	169.7167	2	1000	2405.693	0.137529767
K_1	2.1	75	68.66241	2.286630	0.4373248	0.3	218.8428	2	1050	2452.389	2.081261611
	2.2	88	68.66241	2.286630	0.4373248	0.3	218.8428	2	1100	2452.389	4.162523222
	2.4	70	68.66241	2.286630	0.4373248	0.3	218.8428	2	1200	2452.389	8.325046443
$α_1$	1.1	79	68.66241	2.286630	0.4373248	0.3	218.8428	2	1861.000	3264.034	35.86617321
	1.2	90	68.66241	2.286630	0.4373248	0.3	218.8428	2	3465.724	4868.113	102.633341
	1.3	93	68.66241	2.286630	0.4373248	0.3	218.8428	2	6451.950	7854.339	226.386848