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An Optimal Production Policy Model Considering Defective Items in the Context of Carbon Tax Scheme

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Abstract Carbon tax scheme is one main low-carbon policy to curb carbon emission. This paper develops an optimal production policy model with defective items in a single production period under carbon tax scheme. We assume that the demand for perfect items is price-sensitive and environment-sensitive and the defective items can always be sold. Firstly, we get the optimal production policy and the maximum expected profit. Secondly, we analyze the effect of some parameters emphatically and the results show that when the tax rate is low, firm can’t utilize the optimal production policy unless the rate of perfect product is high. We also find that the optimal production lots may increases when the rate of perfect product increases and the profit may decreases in the same situation, which is inconsistent with intuition.

Keywords: *carbon tax scheme, optimal production policy, defective item, price-sensitive, environment-sensitive*

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1. Introduction

Carbon emission have directly led to global warming, which is noted all over the world. Therefore, Many countries and regions have started to implement low-carbon policies to prevent the process of global warming and protect environment [1]. Carbon tax policy and carbon cap-and-trade policy are the two main policies among them and widely used. Under carbon cap-and-trade scheme, firm is given a maximal emission allowance, which called emission cap. Moreover, this allowance can be traded on the carbon trading market. Carbon cap-and-trade policy can directly reduce the carbon emission of firm while carbon tax policy is an indirect way to reduce the carbon emission. Under carbon tax scheme, firm is charged for each unit of carbon emission at a fixed carbon tax rate. Carl and Fedor [2] investigate the current use of public revenues generated through both carbon tax and cap-and-trade system, cap-and-trade system generated $6.57 billion in total public revenues while carbon tax system generated $21.7 billion. This result proved the economic effectiveness of carbon tax policy and confirmed the widely usage of carbon tax policy emphatically. When firm under carbon tax policy make production decision, carbon tax rate should also be considered besides some traditional factors like cost and price. Previous literature has studied a lot on this topic. Meng et al. [3] develop the optimal strategies for product of two competitive firms with Nash and Stackelberg game structures in the presence of carbon tax. Wu et al. [4] analyze how to determine the production mode under carbon tax with the objective of profit optimization. While He et al. [5] examine the production lot-sizing issues of a firm under carbon tax scheme.

Traditional newsvendor model or economic order model always considers the situation that all products are of perfect quality. However, in the actual process of production, because of the existing of equipment fault, deficiency of quality supervision, limitation of production technology and some other man-made or random factors, defective items are unavoidable. Some literature also take this condition into consideration. Shih [6] considers a model within a production context and assumes that the proportion of defective items is random with a known probability distribution and defective items are returned to the manufacturer at no cost. Papacristos and Konstantaras [7] establish an EOQ model under the items of imperfect quality based on the research of Salamah and Jaber [8], they assume that the shortage of perfect products are not allowed and defective products will be sold at a low price. Yadav et al. [9] develop a supply chain model with defective items while the demand is price-sensitive. A typical assumption in literature is that defective items are reworked or removed from inventory. However, in some situations, defective items cannot be reworked or returned to the supplier. It may then be possible to sell items of imperfect quality at a low price [10]. The latter condition is what we consider in this paper. What’s more, the perfect rate of product is considered as a known probability distribution, which is same as Shih [6].

The above two topics are widely researched, but the literature on the combination of them is relatively limited. This paper considers a one period optimal production policy model with both defective item and tax carbon scheme. Moreover, the demand of perfect item is price-sensitive and environment-sensitive. The rest of the paper is organized as follows: in section 2, we develop the optimal production policy model and get the result of the model. In section 3, we analyze the result and some managerial insights are given. In section 4, we provide concluding comments.

2. Model Description and Formulation

In this section, we describe the condition of our model in reality firstly, then some main assumptions and parameters are given. Finally, we develop the optimal production model and get the optimal production policy.

2.1. Model Description and Assumptions

We consider a setting where a firm makes production decision in a single production period. The items of defective quality will present randomly and defective items will be sold out at a low price r_B, the proportion of perfect item is p, where p is a random variable among [0,1] called the rate of perfect product, we use beta distribution to represents the distribution of p cause it is highly suited for practical purposes especially in the absence of complete data [6]. The firm charged for a tax rate of t for per unit of carbon emission. The demand of perfect item is sensitive to the price r_A and the consumer's environmental awareness θ, which can be represented as $d = \alpha - \delta r_A - \theta e$, where δ is the maximum market demand for perfect product, and θ is the sensitivities of price and consumer's environmental awareness respectively, e is the carbon emission per product. The firm's objective is to determine the optimal production quantity q to maximize the expected profit $\prod$ in one production period. Other notations and assumption are shown as follow:

1) The cost of per unit is c and $r_A > r_B > c$.

2) The quantity of perfect product and defective product are q_A and q_B respectively.

3) The beta distribution is shown as $\beta(a,b)$, which implies that the firm can predicts the expected by the history production performance data.

4) All the parameters mentioned above are larger than zero.

5) $g(\bullet)$ and $G(\bullet)$ are the destiny function and distribution function of p respectively.

2.2. Model Formulation and Solution

Let $E[q_A]$ and $E[q_B]$ denote the expected amount of perfect item and defective item respectively, we can obtain that $E[q_A] = \int_0^1 pqg(p)dp = qE[p]$ and $E[q_B] = \int_0^1 (1-p)qg(p)dp = q(1-E[p])$, what's more, the price of perfect product can be represented as $r_A = \dfrac{\alpha - qE[p] - \theta e}{\delta}$. Hence, the maximum expected profit equation can be given as:

$$\prod(q) = -\frac{E[p]^2}{\delta}q^2 + ((\frac{\alpha-\theta e}{\delta} - r_B)E[p] + r_B - et - c)q \quad (1)$$

The first derivatives of Eq. (1) is $\prod'(q) = -2q\dfrac{E[p]^2}{\delta} + (\dfrac{\alpha-\theta e}{\delta} - r_B)E[p] + r_B - et - c$ and the second derivatives of Eq. (1) is $\prod''(q) = -2\dfrac{E[p]^2}{\delta} < 0$, which imply that the maximum expected profit equation is a convex function of q and the optimal production policy can be driven by

$$\prod'(q) = -2q\frac{E[p]^2}{\delta} + (\frac{\alpha-\theta e}{\delta} - r_B)E[p] + r_B - et - c = 0.$$

So the optimal production policy is:

$$q = \frac{E[p](\alpha - \theta e - \delta r_B) + \delta(r_B - et - c)}{2E[p]^2} \quad (2)$$

Then we can obtain the total carbon emission E by $E = eq$, which is obtained as:

$$E = \frac{E[p]e(\alpha - \theta e - \delta r_B) + \delta e(r_B - et - c)}{2E[p]^2}. \quad (3)$$

The price of perfect product is:

$$r_A = \frac{E[p](\alpha - e\theta + \delta r_B) + \delta(c + et - r_B)}{2\delta E[p]} \quad (4)$$

Finally, the maximum expected profit is obtained as:

$$\prod = \frac{(E[p](-\alpha + \theta e + \delta r_B) + \delta(et + c - r_B))^2}{4E[p]^2\delta} \quad (5)$$

3. Analysis and Discussion

In this section, we analyze the results of the theoretical models in Section 2 and discuss their managerial implications.

Proposition 1. There are two conditions when the optimal production policy is available. That is $t_1 < t < t_2$ when $E[p] < \dfrac{\delta(r_B - c)}{\alpha - \theta e + \delta r_B}$ or $0 < t < t_2$ when $E[p] \geq \dfrac{\delta(r_B - c)}{\alpha - \theta e + \delta r_B}$, where

$$t_1 = \frac{-E[p](\alpha - e\theta + \delta r_B) + \delta r_B - \delta c}{\delta e}$$

and $t_2 = \dfrac{E[p](\alpha - e\theta - \delta r_B) + \delta r_B - \delta c}{\delta e}$. which means that when the carbon tax rate is low, the optimal production policy is more possible to be unavailable unless the rate of perfect product is high. Firm can use the history production performance data to make adjustment.

Proof: The optimal production lots and the price of perfect product should larger than zero when the optimal production policy is efficient. So the numerator of Eq. (2) and Eq. (4) should larger than zero, that is $E[p](\alpha-\theta e-\delta r_B)+\delta(r_B-et-c)>0$ and $E[p](\alpha-e\theta+\delta r_B)+\delta(c+et-r_B)>0$, then we can obtain the range of t is

$$\frac{-E[p](\alpha-e\theta\text{-}\delta r_B)+\delta r_B-\delta c}{\delta e} < t < \frac{E[p](\alpha-e\theta+\delta r_B)+\delta r_B-\delta c}{\delta e}.$$

Let $t_1=\dfrac{-E[p](\alpha-e\theta+\delta r_B)+\delta r_B-\delta c}{\delta e}$ and

$$t_2=\frac{E[p](\alpha-e\theta\text{-}\delta r_B)+\delta r_B-\delta c}{\delta e},$$

when $E[p]<\dfrac{\delta(r_B-c)}{\alpha-\theta e+\delta r_B}$, $t_1>0$, so $t_1<t<t_2$; when $E[p]\ge\dfrac{\delta(r_B-c)}{\alpha-\theta e+\delta r_B}$, $t_1\le 0$, so $0<t<t_2$.

The result of proposition 1 is consistent with intuition. When the history performance of production is good, the firm can realize the optimal production policy under a wider range of carbon tax rate, which implies that high rate of perfect product can provide a higher possibility for firm to achieves the optimal production policy.

Proposition 2. Optimal production lots q is increases with δ, r_B and decreases with θ, e and t. optimal production lots q is increases with the expected rate of perfect product when $t_3<t<t_2$ and

$$0\le E[p]<\frac{2\delta(et+c-r_B)}{\alpha-\theta e-\delta r_B},$$

q is decreases with the expected rate of perfect product when $\max(0,t_1)<t\le t_3$ or $t_3<t<t_2$ and $\dfrac{2\delta(et+c-r_B)}{\alpha-\theta e-\delta r_B}\le E[p]\le 1$, where $t_3=\dfrac{r_B-c}{e}$.

Proof: The first theorem in proposition 2 can be verified directly. Now we demonstrate the second one. The derivative of q with respect to $E[p]$ is:

$$q'(E[p])=\frac{E[p](-\alpha+\theta e+\delta r_B)+2\delta(et+c-r_B)}{2E[p]^3} \quad (6)$$

When $\max(0,t_1)<t\le\dfrac{r_B-c}{e}$, Eq. (6) ≤ 0, q is decreases with $E[p]$. when $\dfrac{r_B-c}{e}<t<t_3$, if $\dfrac{2\delta(et+c-r_B)}{\alpha-\theta e-\delta r_B}\le E[p]\le 1$, Eq. (6) ≤ 0, q is decreases with $E[p]$; if $0\le E[p]<\dfrac{2\delta(et+c-r_B)}{\alpha-\theta e-\delta r_B}$, Eq. (6) >0, q is increases with $E[p]$.

The result of proposition 2 is inconsistent with intuition. We always believe that when the rate of perfect product is high, the optimal production lots will be low. But in some conditions, for example, when the carbon tax is low, the optimal production lots will increases with the expected perfect rate of product. Figure 1 illustrates the phenomenon with numerical results. Another main found is that the optimal production lots will decreases with the consumer environmental awareness.

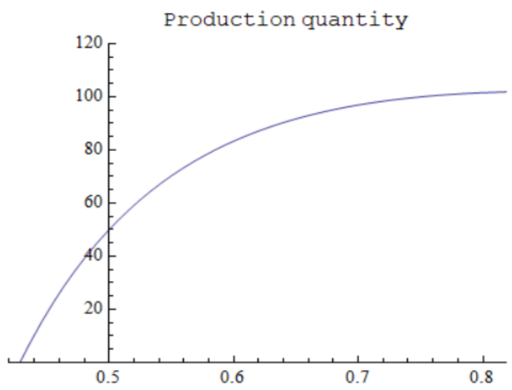

Figure 1. Change in q with respect to $E[p]$ with a low tax rate

Proposition 3. The change in the total carbon emission E with θ, δ, r_B, t and $E[p]$ is same as the optimal production lots q. When t and $E[p]$ satisfy the conditions in proposition 1, if

$$0\le e<\frac{\alpha E[p]+\delta r_B(1-E[p])-\delta c}{2\theta E[p]+2\beta t},$$

E is increases with e; if

$$\frac{\alpha E[p]+\delta r_B(1-E[p])-\delta c}{2\theta E[p]+2\delta t} \le e<\frac{\alpha E[p]+\delta r_B(1-E[p])-\delta c}{\theta E[p]+\delta t},$$

E is decreases with e.

Proof: The first theorem in proposition 3 can be verified directly. Now we demonstrate the second one. Eq. (3) can be rewritten as:

$$E=\frac{-e^2(\theta E[p]+\delta t)+e(\alpha E[p]+\delta r_B(1-E[p])-\delta c)}{2E[p]^2} \quad (7)$$

The increase or decrease of Eq. (7) is determined by the numerator. Let

$$y(e)=-e^2(\theta E[p]+\delta t)+e(\alpha E[p]+\delta r_B(1-E[p])-\delta c),$$

cause $y(e)$ always larger than 0, so $y(e)$ is increases with e when

$$0\le e<\frac{\alpha E[p]+\delta r_B(1-E[p])-\delta c}{2\theta E[p]+2\beta t}$$

and decreases with e when

$$\frac{\alpha E[p]+\delta r_B(1-E[p])-\delta c}{2\theta E[p]+2\delta t} \le e < \frac{\alpha E[p]+\delta r_B(1-E[p])-\delta c}{\theta E[p]+\delta t}.$$

The result of proposition 2 is inconsistent with intuition. Most of us believe that the total carbon emission will always increases with the carbon emission per unit of product. However, when the carbon emission per unit of product is high, the total carbon emission will decreases with carbon emission per unit of product. Figure 2 illustrates the phenomenon with numerical results.

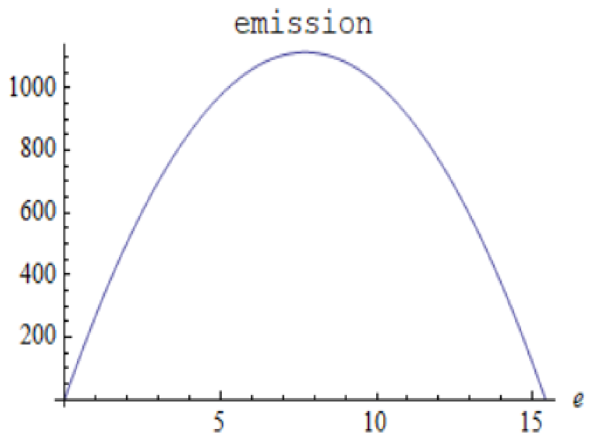

Figure 2. Change in E with respect to e

Proposition 4. The price of perfect product r_A is increases with t and decreases with $E[p]$, θ and r_B. The price of perfect product r_A is increases with e if the price-sensitive is predominant and is decreases with e if the environment-sensitive is predominant.

Proof: The first theorem in proposition 4 can be verified directly. The change of r_A in e is determined by the plus or minus of $\delta t - E[p]\theta$, where t and $E[p]$ satisfy the conditions in proposition 1. Therefore, when the environment-sensitive is predominant in demand function, $\delta t - E[p]\theta$ is negative and r_A is decreases with e; when the price-sensitive is predominant position in the demand function, $\delta t - E[p]\theta$ is positive, r_A is increases with e.

The result of proposition 2 is consistent with intuition. When the consumer environment awareness is high, products with high per unit emission will be unpopular unless the price is low.

Proposition 5. When r_B, θ, t, e and $E[p]$ respectively satisfy the condition that $\max(0,t_1) < t \le t_3$ or $t_3 < t < t_2$ and $\frac{\delta(et+c-r_B)}{\alpha-\theta e-\delta r_B} \le E[p] \le 1$, the expected profit $\prod$ is increases with r_B and decreases with θ, t, e and $E[p]$.; When r_B, θ, t, e and $E[p]$ respectively satisfy the condition that $t_3 < t < t_2$ and $0 \le E[p] < \frac{\delta(et+c-r_B)}{\alpha-\theta e-\delta r_B}$, the expected profit $\prod$ is decreases with r_B and increases with θ, t, e and $E[p]$, where $t_1 = \frac{-E[p](\alpha - e\theta+\delta r_B)+\delta r_B-\delta c}{\delta e}$, $t_2 = \frac{E[p](\alpha - e\theta-\delta r_B)+\delta r_B-\delta c}{\delta e}$ and $t_3 = \frac{r_B - c}{e}$.

Proof: The derivative of $\prod$ with respect to θ, t, e and $E[p]$ are:

$$\prod'(r_B) = \frac{(E[p]-1)\begin{bmatrix}-E[p](-\alpha+\theta e+\delta r_B)\\ +\delta(et+c-r_B)\end{bmatrix}}{2E[p]^2} \quad (8)$$

$$\prod'(\theta) = \frac{e(-E[p](-\alpha+\theta e+\delta r_B)+\delta(et+c-r_B))}{2\delta E[p]} \quad (9)$$

$$\prod'(t) = \frac{e(-E[p](-\alpha+\theta e+\delta r_B)+\delta(et+c-r_B))}{2E[p]^2} \quad (10)$$

$$\prod'(e) = \frac{(\delta t+E[p]\theta)\begin{bmatrix}-E[p](-\alpha+\theta e+\delta r_B)\\ +\delta(et+c-r_B)\end{bmatrix}}{2E[p]^2\delta} \quad (11)$$

$$\prod'(E[p]) = -\frac{(et+c-r_B)\begin{bmatrix}E[p](-\alpha+\theta e+\delta r_B)\\ +\delta(et+c-r_B)\end{bmatrix}}{2E[p]^3} \quad (12)$$

For Eq. (8), when r_B satisfy the condition that $\max(0,t_1) < t \le t_3$ or $t_3 < t < t_2$ and

$$\frac{\delta(et+c-r_B)}{\alpha-\theta e-\delta r_B} \le E[p] \le 1,$$

Eq. (8) is larger than zero, $\prod$ is increases with r_B; when r_B satisfy the condition that $t_3 < t < t_2$ and $0 \le E[p] < \frac{\delta(et+c-r_B)}{\alpha-\theta e-\delta r_B}$, Eq.(8) is less than zero, $\prod$ is decreases with r_B. For Eq. (9), when θ satisfy the condition that $\max(0,t_1) < t \le t_3$ or $t_3 < t < t_2$ and $\frac{\delta(et+c-r_B)}{\alpha-\theta e-\delta r_B} \le E[p] \le 1$, Eq. (8) is less than zero, $\prod$ is decreases with θ; when θ satisfy the condition that $t_3 < t < t_2$ and $0 \le E[p] < \frac{\delta(et+c-r_B)}{\alpha-\theta e-\delta r_B}$, Eq.(8) is larger than zero , $\prod$ is increases with θ. The proof of t, e and $E[p]$ is same as θ.

The result of proposition 5 is inconsistent with intuition. When the carbon tax rate is low, higher rate of perfect product is bad for profit, firm will choose to increases the price of defective product to increases profit. Figure 3 illustrates the phenomenon with numerical results. Moreover, the profit may increases with cost-related parameters like the carbon tax rate, which implies that the high carbon tax rate may harms the interests of consumers.

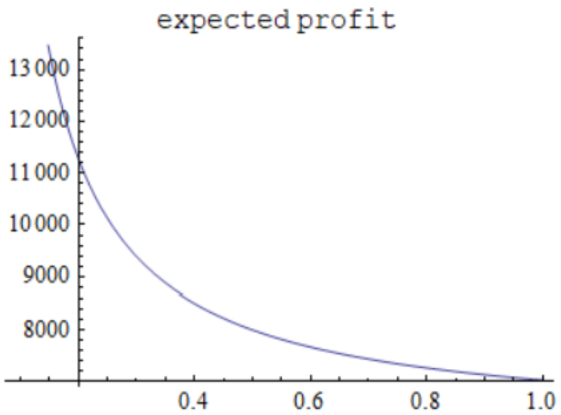

Figure 3. Change in $\prod$ with respect to $E[p]$ with a low carbon tax

4. Conclusion

This paper analyze an optimal production policy model in one single production period considering carbon tax scheme, defective items and consumer environmental awareness. We obtain the optimal production lots with the objective of profit maximization, the total carbon emission, the price of the perfect product and the maximum profit. Then we discuss the relationship of these results with different parameters. Our main finding conclude that when the tax rate is low, firm can't utilize the optimal production policy unless the rate of perfect product is high, which implies that high rate of perfect product can provide a higher possibility for firm to achieves the optimal production policy. The optimal production lots may increases with the expected perfect rate of product while the maximum profit may decreases with the expected rate of product, firm will improve the price of defective product to get more profit. The total carbon emission will decreases with thee carbon emission per product when the carbon emission per product is high, the main reason is the production lots will be constrained in this situation. What's more, the price of defective product has opposite affect on the price of perfect product and when the consumer environment awareness is high, firm should utilizes low-price strategy if the carbon emission per unit of product is high.

This paper has provided useful managerial insight for the production and pricing of the firm. However, there are some limitations that points to the necessity of future research to be performed. The employed model only considers a single production period and ignores the production competition, apparently there is spacious room for model extension. Future research may further consider the issue of multiple period and competition between other manufacturers, moreover, comparison with other low-carbon policy like carbon cap-and-trade policy is also available.

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